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**PILOT/VEHICLE CONTROL OPTIMIZATION USING AVERAGED
OPERATIONAL MODE AND SUBSYSTEM RELATIVE
PERFORMANCE INDEX SENSITIVITIES**

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PILOT/VEHICLE CONTROL OPTIMIZATION USING AVERAGED
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by Gary G. Leininger,* Bruce Lehtinen,[†] and John P. Riehl[‡]

ABSTRACT

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A method is presented for designing optimal feedback controllers for systems having subsystem sensitivity constraints. Such constraints reflect the presence of subsystem performance indices which are in conflict with the performance index of the overall system. The key to the approach is the use of relative performance index sensitivity (a measure of the deviation of a performance index from its optimum value). The weighted sum of subsystem and/or operational mode relative performance index sensitivities is defined as an overall performance index. A method is developed to handle linear systems with quadratic performance indices and either full or partial state feedback. The usefulness of this method is demonstrated by applying it to the design of a stability augmentation system (SAS) for a VTOL aircraft. A desirable VTOL SAS design is one that produces good VTOL transient response both with and without active pilot control. The system designed using the method introduced in this paper is shown to effect a satisfactory compromise solution to this problem.

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1.0 INTRODUCTION

The study of complex systems often involves an investigation into the interconnection of many subsystems and the influence each subsystem has in achieving a prespecified design objective. The optimization of the composite system with respect to a set of adjustable parameters relies upon a knowledge and understanding of the interconnecting structure. Individual subsystem optimization without concern for its cause and effect relation to the composite system may yield an overall system response which deviates substantially from the design specifications. Conversely, a composite system design satisfying the required design objectives might dictate a need for increasingly sophisticated and expensive subsystems. It is, therefore, in the interests of the system designer to have the practical and analytical flexibility to properly align the priorities of competitive subsystems with composite system objectives. A system typifying this design analysis is a pilot/vehicle system.

In the process of designing and evaluating the suitability of a pilot/vehicle system, it is necessary to solicit the pilot's comments and opinion of the handling qualities as one facet of the design. This subjective opinion forms an integral part of the ultimate evaluation of the vehicle and is therefore considered seriously and continuously throughout the design. Optimal performance of the aircraft including the pilot may be in direct conflict with the optimal performance and efficiency of the aircraft when the aircraft is treated as a separate entity. In this respect many studies have been conducted with the objective of mathematically modeling pilot-control characteristics (refs. 1 to 3) and from a practical engineering viewpoint, the development of the quasi-linear model for human pilot dynamics have been one of the beneficial results of these studies (ref. 1). These pilot models can then be used in conjunction with airframe dynamic models in the design of aircraft control systems.

Composite system design can best be achieved when the design criterion includes the evaluations, requirements, and limitations of each individual subsystem. To accomplish this, a generalized theory and design technique is presented. The theory evolves from the concepts and conditions imposed by optimal control theory supplemented by subsystem sensitivity characteristics.

The objective of optimal control theory is to determine the control signals that will cause a process to satisfy the physical constraints and at the same time minimize some performance criterion. In the case of feedback control, the parameters to be optimized are the feedback gains. Once the optimization has been completed, it is natural to inquire into the relative effect of the system response and/or the performance measure to a deviation of the feedback gains from their optimal values. This area of concern is often referred to as "sensitivity."

The design and evaluation of dynamic control systems through the utilization of sensitivity functions has been the subject of intensive research during the past decade. Many different definitions of sensitivity have evolved and system stability, controllability, and other system characteristics have been directly related to these sensitivity functions.

Many analysis, synthesis and optimization techniques used in control theory utilize the sensitivity functions of the state of the system with respect to the system parameters (refs. 4 and 5). These parameter sensitivity functions are often generated by sensitivity models of the system. However, the use of presently available techniques for generating sensitivity functions for linear system containing many parameters results in the simulation of high order dynamic systems. Similarly, control sensitivity (ref. 6), trajectory sensitivity (refs. 4 and 7), eigenvalue sensitivity (ref. 8) and output sensitivity with respect to pole location (ref. 9) all require a simultaneous solution of high order dynamic sensitivity expressions. These sensitivity methods, however, are only intended to account for very small perturbations

from some nominal (or optimal) position. For this reason, most of the above sensitivity functions are not compatible with on-line design, but are primarily used to evaluate the final design. Papers by Cadzow (ref. 10), Dougherty, et al. (ref. 11) and others have applied performance index sensitivity methods to the problem of determining feedback control laws when system parameters are subject to small variations. For larger variations, authors such as Whitbeck (ref. 12), Zadicario and Sivan (ref. 13), and Tuel (ref. 14) have discussed methods for designing controllers which minimize the expected value of the cost functional. All of these techniques are addressed to the plant parameter variation problem as well as a single scalar performance measure.

The emphasis in this paper is directed toward developing a method for designing a practical feedback controller for multivariable linear systems which may be stabilized by output feedback over the entire range of feedback parameters. The distinguishing feature of the proposed technique is the generation of a constant feedback control law subject to the minimization of the performance index sensitivity functions of the composite system and the individual subsystems. This is accomplished by defining a performance index consisting of the sum of relative sensitivity terms of each subsystem multiplied by scalar weighting factors.

Relative sensitivity is a measure of the deviation between the actual value of the performance index and that which would be obtained if the control were optimal, i. e. ,

$$S^R(K) = \frac{J(K) - J(K^0)}{J(K^0)} \quad (1.1)$$

where K^0 is the optimal set of feedback parameters

$$J(K^0) = \min_{K^0} J(K) \quad (1.2)$$

Note that the relative sensitivity is always positive, and thus system performance is always compared with an attainable value.

The performance index of concern here is chosen to be of the form

$$\tilde{J}(K) = \sum_{i=1}^N \lambda_i S_i^R(K) \quad (1.3)$$

$$\sum_{i=1}^N \lambda_i = 1 \quad (1.4)$$

where N is the number of subsystems, $S_i^R(K)$ is the relative sensitivity of the i^{th} system, and λ_i is a weighting factor (or probability factor) associated with the i^{th} subsystem. The performance index of equation (1.3) reflects the interest and concerns of the individual subsystems in the overall decision process. Clearly, small relative sensitivity assures a design close to the optimum and, hence, a smaller influence in the final optimization procedure. Furthermore, the performance measure $J_i(K)$ associated with subsystem i ($i = 1, 2, \dots, N$) need not be of the same form, i. e., quadratic, absolute value, uniform, etc. This, therefore, greatly enhances the design capabilities for large scale systems with subsystem design limitations.

The merit of the performance index of equations (1.3) and (1.4) in the design of practical engineering systems is considered in the next section. Here we restrict the discussion to linear systems with quadratic performance criteria. This restriction enables the designer to utilize the well-developed theory of the optimal linear regulator in establishing the sensitivity terms needed in equation (1.3). The section is then concluded with an example demonstrating the effectiveness of the technique as a useful design tool.

2.0 SENSITIVITY DESIGN FOR OPTIMAL

LINEAR REGULATORS

This investigation is concerned with the design of a feedback control law for a time invariant linear system subject to the minimization of a prespecified scalar performance index. The performance index is chosen in such a manner as to include sensitivity terms associated with subsystems comprising the composite system. Initial consideration is given to the optimization of an individual subsystem with respect to a selected array of feedback parameters. The method is then extended to include several subsystems in the overall optimization procedure using sensitivity concepts derived for the individual subsystem.

Subsystem Optimization

Consider a subsystem whose dynamic performance is characterized by a set of n first order linear time invariant differential equations.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (2.1)$$

where $\mathbf{x}(t)$ is the n dimensional state vector, $\mathbf{u}(t)$ is a vector consisting of m control inputs, and \mathbf{A} and \mathbf{B} are $n \times n$ and $n \times m$ constant matrices describing the system dynamics. The feedback control law

$$\mathbf{u}(t) = -\mathbf{L}\mathbf{x}(t) \quad (2.2)$$

will be optimal if the feedback gain matrix \mathbf{L} is chosen so as to minimize a performance index which is quadratic in the state and control variables

$$J(x, u) = \frac{1}{2} \int_0^{\infty} \{x^T(t)Qx(t) + u^T(t)Ru(t)\} dt \quad (2.3)$$

where weighting matrix Q is positive semidefinite and weighting matrix R is positive definite. The minimization of equation (2.3) will yield a set of constant feedback gains of the form

$$L^* = R^{-1}B^TP^* \quad (2.4)$$

where P^* is a positive definite matrix which is the solution of the steady-state Riccati equation

$$0 = A^TP^* + P^*A + Q - P^*BR^{-1}B^TP^* \quad (2.5)$$

Combining equations (2.4) and (2.5), one obtains

$$0 = (A - BL^*)^TP^* + P^*(A - BL^*) + Q + L^{*T}RL^* \quad (2.6)$$

which is the well-known Lyapunov equation. The resulting value of the performance index when the feedback gain matrix equation (2.4) is substituted into equation (2.2) is

$$J^* = \frac{1}{2} x_0^T P^* x_0 \quad (2.7)$$

The evaluation of P^* from equation (2.5) requires the solution of $n(n+1)/2$ nonlinear simultaneous equations. Alternatively, one could use equations (2.6) and (2.7) in conjunction with a gradient minimization algorithm. For a nonoptimal set of feedback gains L , equation (2.6) becomes

$$0 = (A - BL)^TP + P(A - BL) + Q + L^TRL \quad (2.8)$$

Here A , B , Q , R , and L are known and the $n \times n$ symmetric matrix P can be easily obtained using any of the well-known Lyapunov solving algorithms (refs. 16 and 19). The performance index is evaluated using

$$J = \frac{1}{2} x_0^T P x_0 \quad (2.9)$$

and can be minimized by adjusting the elements of the gain matrix using a gradient minimization algorithm yielding L^* and P^* .

In most practical situations, the initial state of the system is unknown and must be treated as a random vector. Taking the expected value of equation (2.9) yields

$$E\{J\} = \hat{J} = \sum_{i=1}^n \gamma_{ii} p_{ii} \quad (2.10)$$

where p_{ii} are the diagonal elements of the P matrix and γ_{ii} is the covariance of the i^{th} component of the initial state vector with the additional assumption that

$$\gamma_{ij} = E\{x_i(0)x_j(0)\} = 0 \quad (2.11)$$

Note that the minimization of equation (2.10) subject to equation (2.8) will yield an "averaged" set of feedback gains independent of the statistics of the initial state random vector.

The above analysis assumes that the feedback control law $u(t)$ is a linear combination of all the elements of the state vector. In the event that only a select number of the state variables will comprise the feedback control law, equation (2.8) must be modified. Let the p dimensional vector

$$y(t) = Cx(t) \quad (2.12)$$

represent the state variables to be fed back. For a control law of the form

$$u(t) = -Ky(t) = -Kcx(t) \quad (2.13)$$

equation (2.8) becomes

$$0 = (A - BKC)^T P + P(A - BKC) + Q + C^T K^T R K C \quad (2.14)$$

The minimization of equation (2.10) subject to equation (2.14) proceeds as above with the additional restriction that the closed loop system $(A - BKC)$ be stable. This latter restriction is, of course, in effect in the full state feedback system; however, it is well known that the optimal linear state regulator is always stable independent of the open loop dynamics. Clearly, this is not true, in general, for the partial state feedback system and thus one must be cognizant of the location of the closed loop poles, since any solution to equation (2.4) yielding an unstable closed loop system is meaningless.

Relations similar to those of equations (2.8) and (2.14) have been obtained by Kleinman (ref. 17) and Levine (ref. 18), respectively. However, their results are predicated upon the existence of the first partial derivative of equation (2.10) with respect to the unknown feedback gain matrix. Setting this derivative to zero provides the relations upon which their derivation and subsequent results ultimately rely. Consequently, if the feedback gains are constrained in any manner, then $\partial \hat{J} / \partial L \neq 0$ or $\partial \hat{J} / \partial K \neq 0$ at the optimum and the results of (ref. 17) and (ref. 18) no longer apply. However, the constraint boundaries can be incorporated into the gradient algorithm described above and thus minimization of equation (2.10) with respect to the constrained gains can be achieved. The stabilization of $(A - BL)$ for full state feedback and $(A - BKC)$ for partial state feedback is an additional restriction placed upon the feedback gains.

Up to this point a technique has been presented for the optimization of a single subsystem with respect to a set of feedback gains comprising a feedback control law. The method was shown applicable to systems with full state feedback, partial state feedback and both full and partial state feedback with gain constraints. The method will now be extended to include several subsystems with conflicting objectives.

Consider a composite system \mathcal{S}_0 which contains a definable subsystem of interest \mathcal{S}_2 . Two quadratic performance indices, \hat{J}_0 and \hat{J}_2 , are defined for \mathcal{S}_0 and \mathcal{S}_2 , respectively. Each index has been suitably averaged over the initial conditions (as was done in eq. (2.10)). For ease of discussion, assume the system is structured as follows (see figs. 1 and 2).

$$\begin{array}{c}
 \text{System } \mathcal{S}_0 \\
 \hline
 \left. \begin{aligned}
 \dot{\mathbf{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ B_1 & B_2 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\
 \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\
 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} &= \begin{bmatrix} -K_1 & 0 \\ 0 & -K_2 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \\
 \min_{K_1, K_2} \hat{J}_0(x_2, u_1, u_2) &\triangleq \hat{J}_0^*
 \end{aligned} \right\} \quad (2.15)
 \end{array}$$

Subsystem \mathcal{S}_2 (System \mathcal{S}_0 with $C_1 = 0$)

$$\left. \begin{aligned} \dot{x}_2 &= A_{22}x_2 + B_2u_2 \\ y_2 &= C_2x_2 \\ u_2 &= -K_2y_2 \\ \min_{K_2} \hat{J}_2(x_2, u_2) &\triangleq \hat{J}_2^* \end{aligned} \right\} \quad (2.16)$$

Note that both \hat{J}_0 and \hat{J}_2 are functions of subsystem states (x_2) only. Let K_1^0 and K_2^0 be the values of K_1 and K_2 that minimize \hat{J}_0 , and let K_2^2 be the value of K_2 that minimizes \hat{J}_2 . In most physical situations $K_2^0 \neq K_2^2$. This fact reflects a degradation in the performance of subsystem \mathcal{S}_2 when the gains K_2^0 that optimize system \mathcal{S}_0 are used. This degradation can be measured using relative sensitivity, i. e. ,

$$s_{2R}(K) = \frac{\hat{J}_2 - \hat{J}_2^*}{\hat{J}_2^*} \quad (2.17)$$

where \hat{J}_2^* is the value of the performance index for subsystem \mathcal{S}_2 when the feedback gains K_2^2 are employed. A similar expression for the relative sensitivity of \hat{J}_0 is

$$s_{0R}(K) = \frac{\hat{J}_0 - \hat{J}_0^*}{\hat{J}_0^*} \quad (2.18)$$

where \hat{J}_0^* is the value of \hat{J}_0 at K_1^0, K_2^0 . Clearly, $S_2^R = 0$ if $K_2 = K_2^0$, and $S_0^R = 0$ if $K_1 = K_1^0$ and $K_2 = K_2^0$. Consequently, a composite performance index can be defined which incorporates equations (2.17) and (2.18).

$$\hat{J}_3 = \lambda S_2^R(K) + (1 - \lambda) S_0^R(K) \quad (2.19)$$

where λ is a weighting (or probability) factor $0 \leq \lambda \leq 1$. For $\lambda = 0$, the minimization of equation (2.18) will yield the gains K_1^0 and K_2^0 , while for $\lambda = 1$ the gains K_2^0 will result and K_1 will have no effect in the minimization. Thus, for λ in the range of zero to one the minimization of equation (2.18) will result in a tradeoff between the design objectives of subsystem \mathcal{S}_2 and system \mathcal{S}_0 . The appropriate value of λ will depend entirely upon the physical, as well as design, requirements of both the individual subsystem and the composite system.

To generalize the above technique, consider a composite system \mathcal{S}_0 , and N definable subsystems \mathcal{S}_i , $i = 1, 2, \dots, N$, which may be coupled either through state or through control. For each subsystem, define a relative sensitivity of the form

$$S_i^R(K) = \frac{\hat{J}_i - \hat{J}_i^*}{\hat{J}_i^*} \quad (2.20)$$

where \hat{J}_i^* is the value of the performance index when (sub)system i is optimized independent of the rest of the system. The relative performance index can then be formulated as a linear combination of the subsystem sensitivity functions

$$\hat{J} = \sum_{i=0}^N \lambda_i S_i^R(K) \quad (2.21)$$

$$\sum_{i=0}^N \lambda_i = 1 \quad (2.22)$$

where λ_i is the weighting factor associated with the i^{th} (sub)system. The minimization of equation (2.21) will yield a set of feedback gains for the composite system which will provide satisfactory overall performance, while maintaining subsystem response within the design (and economic) specifications. To demonstrate the effectiveness of the proposed technique as a suitable design tool, as well as to illustrate the approach, an example is now presented.

3.0 EXAMPLE

Consider the problem of designing the stability augmentation system (SAS) for a turbojet/lift fan powered VTOL aircraft. Figure 3 is the block diagram for a linearized pitch axis model of a typical VTOL being controlled by a pilot in the hover mode. The states considered in the VTOL model are pitch angle, pitch rate, and acceleration produced by the moment generated by the lift fans. To be designed are SAS gains K_θ , $K_{\dot{\theta}}$, and $K_{\ddot{\theta}}$ such that pitch angle θ is kept close to zero using reasonable amounts of control, u . The complete system is similar in form to the one depicted in figure 2, where the VTOL pitch dynamics comprise the primary subsystem of interest. In this example, C_1 reflects the fact that only one pilot state (δ) is measurable; and for simplicity, the gain K_δ associated with this state was fixed.

Pilot dynamics are described by the third order model shown in the diagram (ref. 20). Pilot parameters are pilot gain K_p , lead time constant τ_L , muscle lag τ_M , sensor lag τ_S , and pilot dead time τ_D (as a Pade approximation). Aircraft handling qualities studies have shown that if the parameters the pilot adaptively adjusts (K_p and τ_L) are not too large, the pilot will give the aircraft a high rating. For

this example, K_p was fixed at 13.5 cm/rad and τ_L was fixed at 0.5 sec. These are relatively low values; hence the resulting design should get a good pilot rating. Fixed pilot parameters were assumed to be $\tau_S = 0.062$ sec, $\tau_M = 0.36$ sec, and $\tau_D = 0.35$ sec. As indicated previously, stick sensitivity K_δ was not optimized, but was chosen to be 0.6 rad/sec²/in., based on typical pilot preferences.

The VTOL dynamic model includes parameter τ_1 which represents the lag between stick deflection and pitching moment produced by the engine/lift fan combination. τ_1 was assumed to be 0.3 second in this study, since experience has shown that pilot may have difficulty in controlling the system if the actuation lag is greater than this amount. Conversely, it is desirable, as far as lift fan/engine design is concerned, to have τ_1 as large as possible.

The VTOL can be operated in either of three modes: (1) pilot-in-the-loop (PIL) where both pilot and SAS contribute to stabilization ($K_p \neq 0$), and (2) pilot-out-of-the-loop (POL) where $K_p = 0$ and all stabilization derives from the SAS, and (3) SAS-failed mode, where the pilot provides all stabilization. Because of these three modes of possible operation, a conflict arises in designing the SAS and stick sensitivity K_δ . It was decided that K_δ would not be optimized in this study, hence, the SAS-failed mode has been ignored. For the PIL and POL modes, SAS optimized for the POL mode may produce a system too insensitive to the pilot's control during PIL mode operation. On the other hand, a SAS which is designed to be optimal when the pilot is in the loop may not sufficiently stabilize the aircraft in the fixed-stick (POL) mode. For this example, the problem of conflicting performance objectives was solved by using a composite performance index

$$\hat{J}_3 = \lambda \left(\frac{\hat{J}_2 - \hat{J}_2^*}{\hat{J}_2^*} \right) + (1 - \lambda) \left(\frac{\hat{J}_0 - \hat{J}_0^*}{\hat{J}_0^*} \right) \quad (3.1)$$

Here, \hat{J}_2 is an index of performance for POL operation, \hat{J}_0 corresponds to PIL operation, and λ is a weighting factor. Lambda could

be, for example, the probability of the aircraft being flown in the POL mode. The starred \hat{J} values are those that are obtained when optimizing for PIL or POL mode operation separately.

The form of the performance index chosen to be minimized in each mode of operation is

$$\hat{J} = E\{J\} = E\left[\int_0^{\infty} (\theta^2 + k_{\delta}\delta^2 + k_u u_{SAS}^2) dt\right] \quad (3.2)$$

where k_{δ} and k_u are scalar weighting constants and \hat{J} is to be averaged over the initial states. The three terms in this performance index were selected in accordance with the following considerations: (1) θ should be driven to zero as rapidly as possible, (2) required pilot stick deflection should not be excessive, (3) a control moment command, u_{SAS} , generated by the SAS should not cause the lift fans to exceed their rated thrust. For POL operation, the aircraft performance index can be written in the form of equation (2.10) as

$$\hat{J}_2 = \sum_{i=1}^3 \lambda_{ii} p_{ii} \quad (3.3)$$

where the P matrix for the aircraft subsystem is obtained by solving a third order equation of the form of equation (2.14). Similarly, for the pilot-in-the-loop,

$$\hat{J}_0 = \sum_{i=1}^6 \gamma_{ii} p_{ii} \quad (3.4)$$

Here P is the solution of equation (2.14) for the complete sixth order system with the stipulation that $\gamma_{ii} = 0$ for the three pilot states (initial pilot states are assumed to have zero mean and variance). For

both modes, initial states are assumed to be uncorrelated ($\gamma_{ij} = 0$, $i \neq j$).

Solutions were obtained (optimal SAS gains) for various values of λ . Powell's method (ref. 21) of function minimization was used along with the Lyapunov equation solution technique of reference 16 for evaluation \hat{J} . Covariances γ_{11} , γ_{22} , and γ_{33} were all assumed to be 1.0, and weighting factors k_δ and k_u were chosen as 0.0015 and 0.15, respectively. Figure 4 graphically presents the results of the optimization of the composite performance index, \hat{J}_3 , as a function of SAS pitch gain K_θ for a selection of λ values. Each curve is a section through the performance surface with K_δ and K_u held constant at the optimum values obtained for that particular λ value. Optimal K_θ 's (which occur at the minima) range from 2.19 to 3.69; however, their magnitudes are quite similar for the extreme cases ($\lambda = 0$ and 1). The design trade-off is evidenced by the fact that \hat{J}_3 increases as λ moves away from 0 or 1, up to a maximum of about 0.04 for the $\lambda = 0.6$ curve.

To further demonstrate the influence of λ on control system behavior, typical transient responses for fixed initial conditions were computed. Figures 5(a) and (b) show, respectively, PIL and POL responses of pitch angle θ , moment command due to the pilot u_p , moment command due to the SAS u_{SAS} , and the resulting VTOL pitch acceleration $\ddot{\theta}$. Transients are displayed for four λ values (0, 0.2, 0.8, and 1.0) for zero pilot initial conditions and VTOL initial conditions of 0.1, 0.1, and 0.1.

Comparing the pitch angle transients, it can be seen that for PIL operation (fig. 5(a)), the best transient occurs for the system optimized for $\lambda = 0$. Conversely, the best POL transient (fastest response) occurs in figure 5(a) for the system optimized for $\lambda = 1$. As an example, consider the case when the VTOL is in the POL mode 80 percent of the time, i.e., $\lambda = 0.8$. The θ curve for $\lambda = 0.8$ in figure 5(a) shows performance is somewhat degraded (higher overshoot, poorer damping) over the $\lambda = 0$ case, but is not nearly so poor as the highly underdamped $\lambda = 1$ case. In figure 5(b) it can be seen that the $\lambda = 0.8$

design is very nearly as fast responding as the $\lambda = 1$ design, and much faster than the system designed for PIL operation ($\lambda = 0$).

Similar comparisons can be made for u_p , u_{SAS} , and $\ddot{\theta}$ transients. For instance, pilot control excursion is high in figure 5(a) for the $\lambda = 1$ case, (PIL operation with POL feedback gains). However, using these same gains in POL operation gives the best transient performance. Note that u_p is zero in figure 5(b), since the pilot is not exercising control. Pitch acceleration histories are included to demonstrate that all of the optimal controllers give rise to VTOL accelerations which are "reasonable" in magnitude. The rather anomalous behavior of u_p in PIL operation (fig. 5(a)) at $t = 0$ is due to the fact that in the analysis and transient calculation, pilot dead time has been approximated by a first order Pade approximation. What appears to be the pilot initially attempting to increase the error in θ is actually due to the inaccuracy in modeling his dead time with a Pade.

In the preceeding example, no constraints were imposed on the SAS gains. One obvious constraint that could be considered is one on $K_{\dot{\theta}}$. As $K_{\dot{\theta}}$ becomes large, the lift fan/engine eigenvalue increases, such that eventually saturation will certainly occur. Thus, for a reasonable solution, $K_{\dot{\theta}}$ must be bounded. Another problem, mentioned in Section 2, is system stability. The system in this example was open loop stable such that even though not all states were fed back, a set of (optimal) gains were found which produced a stable system. This will not be the case, in general, so that periodic stability checks must be made during the optimization to insure each set of (sub-optimal) gains corresponds to a stable system.

4.0 CONCLUSIONS

An approach has been formulated to the problem of designing a control for a system with conflicting subsystem performance indices. Use was made of relative sensitivity by introducing it into the system's performance index. A method was developed for handling linear systems

with quadratic subsystem performance indices, for either full or partial state feedback. The approach was demonstrated by using it to design the pitch axis SAS for a piloted VTOL, where the main subsystem of interest was the VTOL aircraft. A design was obtained, consisting of a fixed set of SAS gains, which gave acceptable performance both with and without pilot control. The methods developed could be extended to include nonlinear plants, state variable constraints, and nonquadratic performance indices. They could also be applied to designing the complete three axis SAS for a VTOL, capable of operating throughout the hover, transition and cruise modes.

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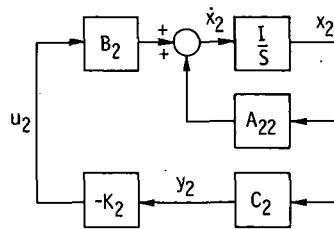


Figure 1. - Subsystem \mathcal{S}_2 isolated from composite system \mathcal{S}_0 .

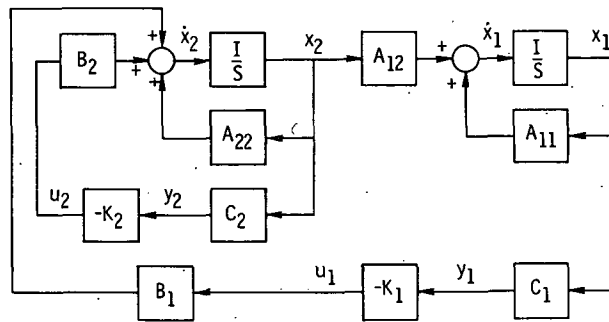


Figure 2. - Composite system \mathcal{S}_0 .

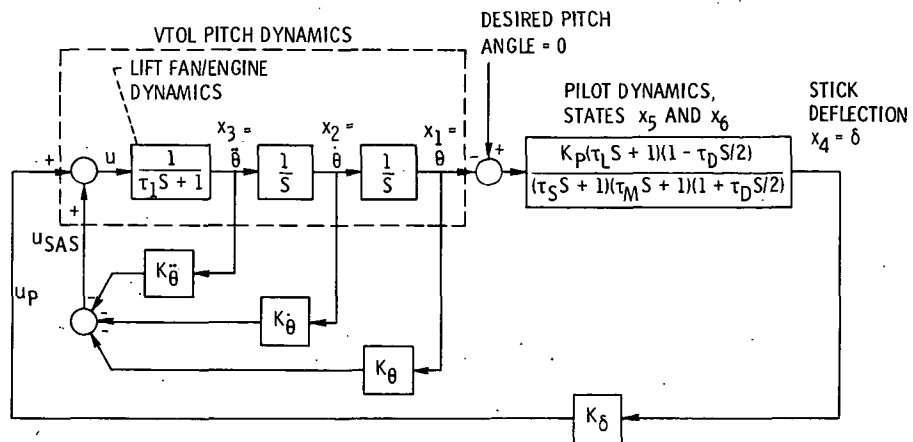


Figure 3. - Block diagram of VTOL aircraft controlled by pilot in hover mode.

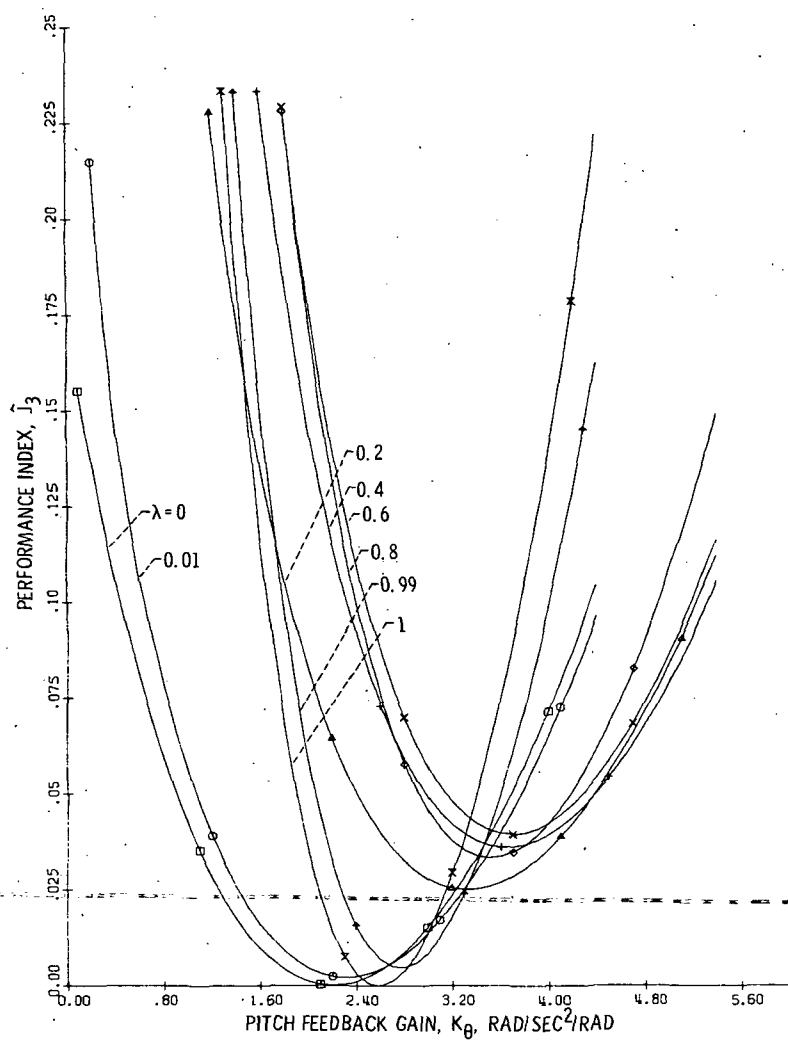
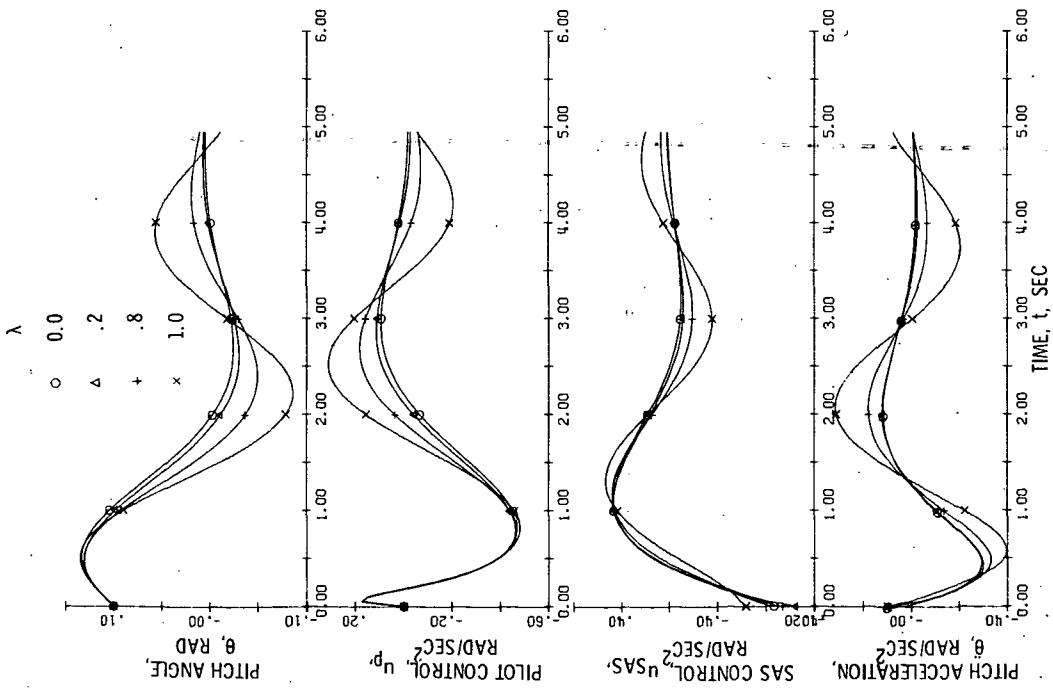
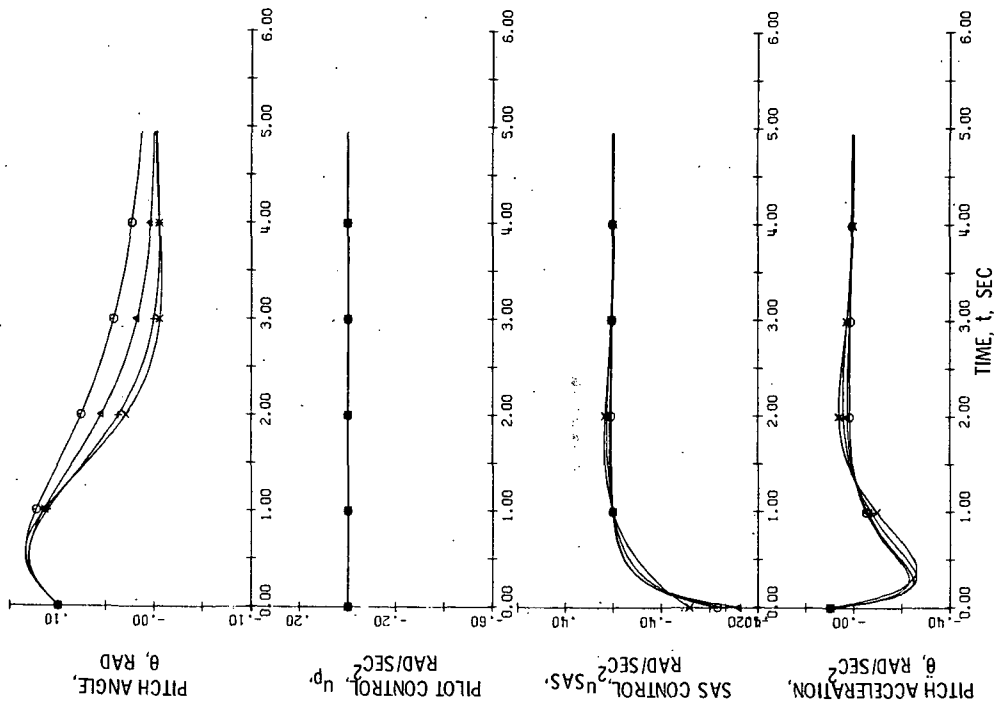


Figure 4. - Performance index \hat{J}_3 as a function of K_θ for K_δ and K_θ kept at optimum values; weighting factor λ as a parameter.



(a) RESPONSES WITH PILOT IN THE LOOP (PIL).

Figure 5. - Transient responses for VTOL and pilot for different sets of optimal gains; initial pilot states equal zero, $\theta(0) = \dot{\theta}(0) = \ddot{\theta}(0) = 0$.



(b) RESPONSES WITH PILOT OUT OF THE LOOP (POL).

Figure 5. - Concluded.